Journal of Applied Mechanics and Technical Physics, Vol. 50, No. 6, pp. 965–971, 2009

RELATIONSHIP BETWEEN SPALL CHARACTERISTICS AND THE DIMENSION OF FRACTAL FRACTURE STRUCTURES

B. K. Barakhtin[1] **and G. G. Savenkov**[2]

UDC 669.017:539.375

The effect of the fractality of a fracture surface and spall contour on the characteristics [fracture time (strength) and spall strength] of the loaded material is studied. It is shown that an increase in the fractal dimensions of the spall contour leads to an increase in the material strength parameter in the tensile wave and spall strength, whereas an increase in the fractal dimension of the fracture surface leads to a decrease in the spall strength. As an example, the spall strength is calculated taking into account the fractality of the fracture surface for Sp. 28 steel.

Key words: *mesodefect, fractal, spall, fracture time, spall strength.*

Introduction. As is known, the dynamic (in particular, spall) fracture of materials is a multiphase kinetic process based on the polyscale evolution of internal material defects. At each scale level, the nature and duration of elementary events of structural changes are different and interrelated. In addition, the initiation and development of defects at different stages is possible, i.e., the formation of macrodefects can be accompanied by the initiation of submicrodefects, their coalescence with the formation of microdefects, etc. In this case, the indicated processes influence each other. From the aforesaid, it follows that constructing a mathematical multilevel model of dynamic (spall) fracture taking into account the multifactor nature of the process (description of each level, determination of the relations between the physical processes at each level, and determination of the influence of the processes occurring at all levels on the processes occurring at a chosen level) is a difficult problem. There are a number of models describing various features of material behavior under dynamic (spall) fracture conditions. Wide use has been made of models that include only two fracture stages (microlevel and macrolevel) or employ only statistical methods for describing the initiation and development of defects [1].

Recently, models has been developed that take into account the discrete nature of a deformable medium, which is treated as a dissipative system, whose evolutions leads to the occurrence of fractal structures capable of self-similar propagation resulting in global fracture upon reaching some critical conditions [2–5].

The purpose of the present work is to establish relations among the strength $t_{\rm fr}$, the spall strength $\sigma_{\rm fr}$ of the material, and the fractal parameters of the fracture surface formed during shock-induced spall fracture.

Experimental Technique. Disks-shaped targets 52 mm in diameter and 5–10 mm thick made of steels of various classes were loaded by a flat impactor using a pneumatic gun at velocities $V_0 = 200$–650 m/sec.

The tested targets were studied with a SEM 535 scanning electron microscope. The fractal dimensions of the spall contour and the size distribution of shear and spall regions (defects) were determined by digitization and statistical processing of electron photomicrographs of vertical sections of the fracture surface of the targets at magnifications over a wide range (from 10 to $5 \cdot 10^3$).

Experimental Results and Discussion. The mechanical parameters of the tested materials have been analyzed previously [4–6]. The data [4–6] of the electron-microscopic scanning of the tested samples show that, for the majority of steel grades (40Kh, 45KhMFBA, 12Kh18N10T, Sp. 28, and others), the fracture surfaces or

[1]Prometei Central Research Institute of Structural Materials, St. Petersburg 191015. [2]Poisk Research Institute, Murino 188662, Leningrad Region; sav-georgij@yandex.ru. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 50, No. 6, pp. 61–69, November–December, 2009. Original article submitted April 8, 2008; revision submitted November 7, 2008.

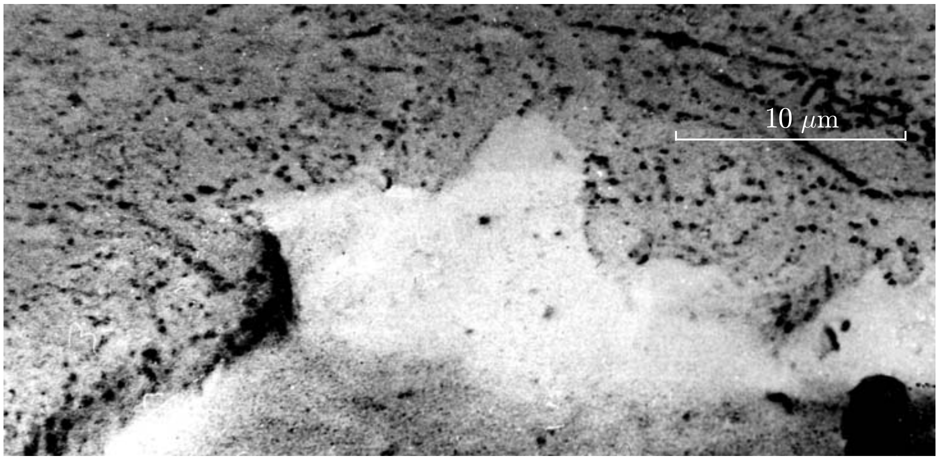

Fig. 1. Element of the spall crack contour in the shape of Koch stars in a Sp. 28 steel sample.

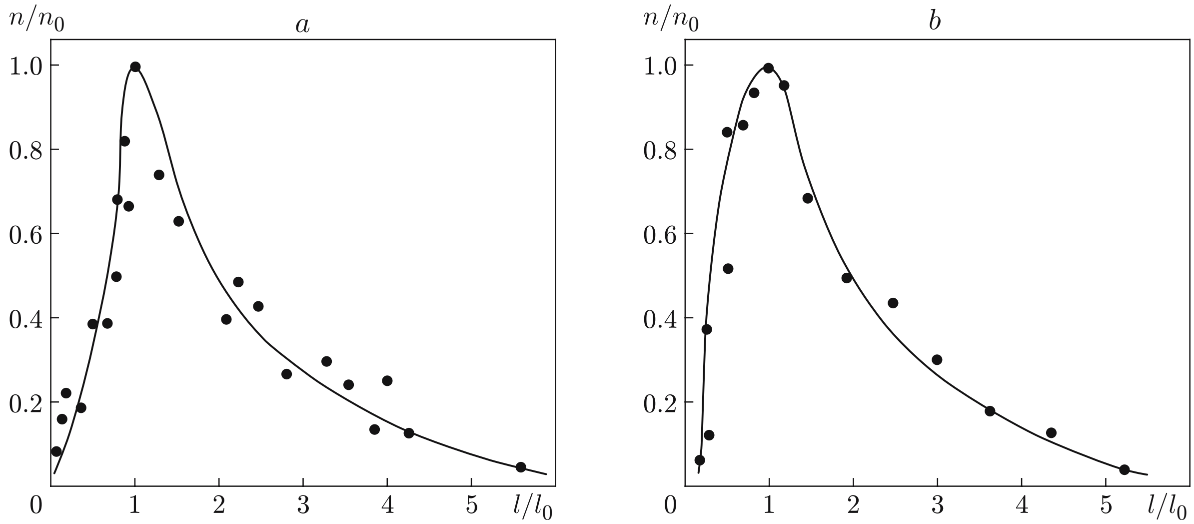

Fig. 2. Defect size distribution in a 40Kh steel sample at $V_0 = 620$ m/sec: (a) spall defects parallel to the free surface of the sample ($n_0 = 324$ and $l_0 = 24$ μm); (b) shear defects perpendicular to the free surface ($n_0 = 199$ and $l_0 = 11$ μm); curves correspond to calculation results, and points to experimental data.

spall crack contours were fractal objects (Fig. 1). Depending on the strength of the target, these contours have the shape of curved lines which can be described by Koch curves [2]. In addition, the size distributions of the shear and spall regions are described by the Weibull) function, for example, in experiments on dynamic loading of 40Kh steel samples ($\sigma_{0.2} = 415$ MPa, $\sigma_{\rm W} = 690$ MPa, and $\delta_5 = 8.5\%$) with an initial impact velocity $V_0 = 620$ m/sec. Figure 2 shows curves of n/n_0–l/l_0 (n_0 is the maximum sum of defects of size l_0 in this distribution). In the case of damage accumulation and brittle fracture, the physical treatment of the Weibull distribution corresponds to the fractal geometry of the defect structure of metal [2, 7]; therefore, it can be assumed that most of the approximating relations describing the development of a crack spall are potentially fractal [3].

Using the spatial classification of structural defects of a real metallic material [8], we determine the range of fractal scales in which manifestations of fracture fractality and self-similarity for spall cracks are the most probable. As is known, in a tensile wave, fracture nuclei of size 0.2–100.0 nm are formed in an explosive manner with the sound velocity [9]. Because in the strain pulse, the time of their formation is small (for steels, less than 0.02 nsec), the effect of the microstructural level on the strength of the sample can be ignored. Thus, the dominant elementary events of fracture occurring in a tensile wave occur at two mesoscale levels: mesolevel I (fractal sizes of 0.1–20.0 μm) and mesolevel II (fractal sizes of 20–2000 μm).

Computation Models. We study the effect of the fractality of a spall contour on the characteristic fracture time in a tensile wave. The total length of the fractal contour of a spall crack is equal to

$$l = \Delta l\, N, \tag{1}$$

where Δl is the minimum distinguished size of a mesodefect responsible for the formation of a fractal spall crack (the scale unit used for step-by-step circulation around the fractal curve) [2]; N is the number of distinguished mesodefects.

The scale Δl is contained in the relation for the fractal dimension D:

$$D = \log\,(l/\Delta l)/\log\,(l_1/\Delta l). \tag{2}$$

Here l_1 is the distance between two extreme points of the macrocrack along the straight line. As is known, a fundamental property of fractal curves is their self-similarity or reproducibility in a certain range of scales, i.e., these curves can have the largest scale, but, in principle, a small scale in which the basic curve is not reproduced should not exist (although restrictions from below take place).

In view of (1), in spall fracture, the strength of the sample in a tensile wave is equal to

$$t_{\rm fr} = \alpha(\Delta l/V_{\rm cr})N + Nt_{\rm d}, \tag{3}$$

where $V_{\rm cr}$ is the growth rate of the mesodefect; $t_{\rm d}$ is the delay time of the start of the mesodefect [10]; the coefficient $\alpha = 0.5$ for metals having pronounced longitudinal mesocrack due to a large velocity dispersion Δu in a wave compression [6], and $\alpha = 1$ for metals for which the velocity dispersion Δu is absent or is very small (in this case, longitudinal mesocrack are not formed).

Relation (3) for the fracture time assumes that spall cracks grow from one center, whereas during spall and fracture, as a rule, many fracture nuclei of size 0.2–100.0 nm are formed (in this case, the average value of the fractal size is closer to the lower limit). It is assumed that the maximum concentration of such submicrodefects formed in time less than 1/30 sec in a metal sample in the prefracture state reaches values of the order of 10^{14}–10^{15} m^{-3} [11]. It has been shown that during spall, the concentration of microdefect nuclei does not exceed 10^8 m^{-2}. Accordingly, the concentration of defects on the line $N_l \approx 10^4$ m^{-1}, i.e., the total initial length of submicrodefects is equal to $l = 2\cdot 10^{-6}$–10^{-3} m (it can be assumed that, on the average, $l = 2\cdot 10^{-5}$–$2\cdot 10^{-4}$ m). In this case, it can be assumed that the spall crack grows from one fracture site with the indicated initial size, and, hence, that relation (3) is valid.

The delay in the start of the mesodefect $t_{\rm d}$ is determined by the duration of the transition process of establishment of stresses in the vicinity of the defect, and in the order of magnitude, it is equal to [10]

$$t_{\rm d} = \Delta l/C, \tag{4}$$

where C is the longitudinal sound velocity.

The scale Δl can be determined from relation (2) (for this, it is first necessary to find the fractal dimension D, for example, using the method of vertical sections [4]) or to specify its concrete value because the dimension depends weakly on the chosen measurement scale unit (a geometrical unit with a constant multiplicity or a physical unit with multiplicity due to structural characteristic scales). The most suitable value $\Delta l = 0.1$ μm, which is the intermediate one between the microlevel and mesolevel I. There is no consensus on the velocity of mesocracks (and macrocracks) $V_{\rm cr}$, but it can be believed that, for high-velocity loading processes, this velocity is close to the maximum crack velocity which is identified with the propagation velocity of Rayleigh waves $C_{\rm R}$, i.e., it can be assumed that $V_{\rm cr} \simeq C_{\rm R}$.

In view of the values of the parameters include in relation (3), the first term of this relation is close to uncertainty of the form of $0\cdot\infty$; at the same time, under spall fracture conditions, the strength of the sample in a tensile wave is a finite quantity (as a rule, not more than 1 μsec). In view of the aforesaid, the existence of a characteristic scale implies that a fractal main crack should propagate in a discrete (jumpwise) manner. Then, the minimum structural unit (quantum of the crack jump [2]) that uniquely determines the properties and structure of such a macrocrack is the minimum distinguished mesodefect of size Δl (scale of a fractal crack). It should be noted that the discrete nature of fracture has been discussed in many papers. For example, in [12], the rupture of the bonds of one pair of atoms of an ideal nonclose-packed cubic lattice is suggested as the quantum of the crack jump.

The time

$$\Delta t = \frac{\Delta l}{V_{\rm cr}} \simeq \frac{\Delta l}{C_{\rm R}}$$

is the time of formation of the minimum distinguished mesodefect of size Δl and, in fact, it is the so-called structural time $t_{\rm s}$ introduced in [13], which is a constant of the material and problem:

$$t_{\rm s} = d/C.$$

In [13], the quantity d is understood as the linear size characterizing a unit fracture cell at the given scale level. It is proposed to determine this linear size from the condition

$$d = 2K_{\rm Ic}^2/(\pi\sigma_c),$$

where $K_{\rm Ic}$ is the critical stress intensity factor and σ_c is the rupture strength of a faultless sample of the material considered. The parameter d is included in the Neuber–Novozhilov static fracture criterion and the structural-time criterion proposed in [14]. Other interpretations of the parameter d have also been proposed, for example: the grain size for a polycrystalline material, the parameter of scale compatibility of strength characteristics, etc.

Since, in the case considered, we study a fractal macrocrack, its length can be defined by the formula

$$l = \lambda(\Delta l)^{1-D}, \tag{5}$$

where λ is a constant. From (1) and (5), it follows that

$$N = \lambda(\Delta l)^{-D}.$$

Substituting this relation into (3) and taking into account (4), we obtain

$$t_{\rm fr} = \alpha(\Delta l/C_{\rm R})\lambda(\Delta l)^{-D} + \lambda(\Delta l/C)(\Delta l)^{-D}.$$

Because, in the range $\nu = 0.28$–0.31 (the most widely used values of Poisson ratio for structural materials), $C_{\rm R} \approx 0.6C$, we finally have

$$t_{\rm fr} = \lambda(1.67 + \alpha)(\Delta l/C_{\rm R})(\Delta l)^{-D}. \tag{6}$$

Relation (6) implies that the larger the fractal dimension D, the larger the fracture time in a tensile wave (strength of the sample). Since the fractal dimension is linked to the relief roughness parameter by the relation [2]

$$D = \log R - \log k/ \log \eta,$$

where $R = l/l_1$ is the roughness parameter of the macrocrack contour, k is a constant, η is the observation scale, it is obvious that the larger the roughness of the fracture contour, the higher the strength. We note that, in the present paper, quantitative relations are obtained for the first time. We also note that the parameter R is an integral characteristic of contour and ignore its irregularity which takes place in practice. This irregularity of course makes a contribution to the characteristic fracture time.

In the determination of the force and energetic criteria for spall processes (spall strength $\sigma_{\rm fr}$ and specific (per unit surface area) spall work $\lambda_{\rm fr}$) in experiments on spall fracture of materials, it is assumed that strictly one-dimensional regular deformation occurs which is equivalent to the ideal restraining of material [15]. However, fractal curves are geometrically chaotic curves; therefore, in the case of a fractal spall crack contour, deformation motions of the material are of random chaotic nature. In view of the aforesaid, we obtain the relationship between the strengths and fractal dimension, which, in fact, is a measure of chaos of the geometrical structure of the crack [16].

The spall strength $\sigma_{\rm fr}$ (critical spall stress) ignoring the elastoplastic properties of the material is found from the well-known relation

$$\sigma_{\rm fr} = \rho_0 C_0(V_0 - V_{\rm min})/2, \tag{7}$$

where ρ_0 is the initial density of the material, C_0 is the bulk sound velocity, V_0 is the maximum velocity of the free surface, and $V_{\rm min}$ is the velocity of the free surface at the first minimum of the dependence $V(t)$.

As noted above, the spall fracture process is a multistage one and a system of defects is formed in the incident loading wave, resulting in a structural change and heating of the target material. Apparently, these factors are responsible for the random nature of motion of particles of the medium in the tensile wave, which ultimately forms the random trajectory of the main macrocrack.

As is known, the physical properties (density and elastic moduli) of a medium with defects depend on the degree of its imperfection; therefore, it should be assumed that formula (7) is applicable to an ideal (or more precisely, initial) material and does not describe its real structure by the moment of rupture [17].

Let us estimate the density ρ, Young modulus E, the bulk compression modulus K, and Poisson ratio ν of the sample material (without spall) with the fractal defect structure formed. A more rigorous approach to the calculation of these characteristics should be based on multifractal formalism, but in the case considered, it is sufficient to use two fractal dimensions: the fracture surfaces and the spall crack contour obtained at the mesoscopic scale-structural level.

The density distribution n_S of all defects on the fractal surface is written as [2]

$$n_S = \beta \langle l \rangle^{-D_S}.$$

In view of this relation, the effective Young elastic modulus and Poisson ratio are given by the following relations [17]:

$$E_{\text{eff}} = E[1 - (\pi/4)\beta\langle l \rangle^{2-D_S}], \qquad \nu_{\text{eff}} = \nu[1 - (\pi/4)\beta\langle l \rangle^{2-D_S}].$$

Here β is a certain constant coefficient and $\langle l \rangle$ is the average size of the mesodefect. The expression for the effective bulk compression modulus, in view of its definition, is written as

$$K_{\text{eff}} = E_{\text{eff}}/[3(1 - 2\nu_{\text{eff}})].$$

To determine the density of the damaged material and the bulk sound velocity, it is necessary to find its volume change δW, which can generally be represented as

$$\delta W = \delta W_e + \delta W_g \pm \delta W_r \tag{8}$$

(δW_e is the volume change due to elastic deformation, δW_r is the volume change due to the stress relaxation during plastic deformation of the material, and δW_g is the volume increase due to defect formation at all structural levels).

The first and third terms of the relation (8) are negligibly small. In classical spall theory assuming that the softening reaches 3–5%, the term δW_g is also ignored [18]. However, in the case considered, accounting for the volume increase due to softening of the material is required to determine the effect of the fractality of the system of mesodefects on the fracture surface on spall characteristics.

In view of the aforesaid, Eq. (8) becomes

$$\delta W \simeq \delta W_g.$$

Along with the roughness parameter of the fracture contour R, we introduce the roughness parameter of the fracture surface $R_1 = S/S_{\text{fr}}$ (S is the true area of the fracture surface and S_{fr} is the area of the corresponding rough surface projected onto the plane). The quantities R and R_1 are linked by the relation

$$R_1 = 1 + (4/\pi)(R - 1). \tag{9}$$

Because $R = \xi\eta^{1-D}$ [2], using relation (9), we obtain

$$\delta W \simeq \delta W_g = S_{\text{fr}}(\xi\eta^{1-D} - 1)\langle l \rangle 4/\pi.$$

As a result, the effective density is given by

$$\rho_{\text{eff}} = \rho_0/[1 + \gamma(\xi\eta^{1-D} - 1)(4/\pi)],$$

where $\gamma = \langle l \rangle/H$ (H is the initial thickness of the sample).

In view of the obtained relations for K_{eff} and ρ_{eff}, the effective bulk sound velocity is equal to

$$C_{0,\text{eff}} = C\{[1 - (\pi/4)\beta\langle l \rangle^{2-D_S}][1 + \gamma(4/\pi)(\xi\eta^{1-D} - 1)]/3[1 - 2\nu(1 - (\pi/4)\beta\langle l \rangle^{2-D_S})]\}^{0.5}.$$

From the relation for $C_{0,\text{eff}}$, it follows that for a value of the coefficient β close to or equal to the value of $(4/\pi)\langle l \rangle^{D_S-2}/\langle l \rangle^2$, the sound velocity is close or equal to zero (sound damping effect). This conclusion is in good agreement with experimental results on the scattering of sound waves by fractal surfaces [19], which is due to the distribution of the slopes of the fractal surface. It should be noted that the problem of sound wave propagation through a system of defects is somewhat similar to the problem of gas penetration through a system of pores in coal gas masks, which promoted the development of percolation theory [20].

TABLE 1

Calculated Spall Strengths Taking into Account the Fractal Dimension of the Fracture Surface in a Sp. 28 Steel Sample

V_0, m/sec	$\beta \cdot 10^2$	$\langle l \rangle$, μm	ξ	D	$(\sigma_{\rm fr})_{\rm exp}$, GPa	$(\sigma_{\rm fr})_{\rm calc}$, GPa
152.5	1.4	12	11.0	1.242	1.37	1.13
196.0	1.3	15	10.5	1.200	1.36	1.25
213.0	1.3	16	10.0	1.188	1.36	1.30

Substituting the effective density and bulk sound velocity into (7), we obtain the following expression for the spall strength:

$$\sigma_{\rm fr} = 0.5\rho_0 C\,\Delta V\,\{[1-(\pi/4)\beta\langle l\rangle^{2-D_S}]/3[1-2\nu(1-(\pi/4)\beta\langle l\rangle^{2-D_S})][1+\gamma(4/\pi)(\xi\eta^{1-D}-1)]\}^{0.5}.$$

Here $\Delta V = V_0 - V_{\rm min}$.

From the above relation it follows that the spall strength of the target material decreases with increasing fractal dimension of the fracture surface, but it increases with increasing fractal dimensions of the spall contour. Table 1 gives the results of calculations performed by the relation for the spall strength for a Sp. 28 steel samples ($E = 204$ GPa, $\nu = 0.28$, $\rho_0 = 7.8 \cdot 10^3$ kg/m^3, and $C_0 = 4450$ m/sec) for various values of the initial impact velocity V_0. The experimental data are taken from [4, 21]; in all cases, the maximum scale of the increase is $\eta = 5 \cdot 10^3$ [4] and the fractal dimension of the fracture surface is $D_S = 1 + D$, $\gamma = 1.5 \cdot 10^{-3}$.

Conclusions. The study allows the following conclusions to be drawn. Accounting for the fractality of fracture surfaces leads to: 1) a decrease in the spall strength; 2) an increase in the spall strength with increasing impact velocities, which is in better agreement with experimental values.

We note that, in comparative spall tests of materials, it is reasonable to find not only the quantities $t_{\rm fr}$ and $\sigma_{\rm fr}$ but also the fractal dimensions of the spall crack contour D and the fracture surface D_S, which determine the indicated properties of the material under dynamic loading. In this case, the methods of determining the indicated fractal dimensions should be specified by regulations because the dimension is known to depend markedly on the measurement method.

REFERENCES

1. B. L. Glushak, I. R. Trunin, S. A. Novikov, and A. I. Ruzanov, "Numerical modeling of spall fracture," in: *Fractals in Applied Physics* [in Russian], Inst. of Exp. Phys., Arzamas-16 (1995), pp. 59–122.
2. V. S. Ivanova, *Synergetrics: Strength and Fracture of Metals* [in Russian] Nauka, Moscow (1992).
3. V. S. Ivanova, A. S. Balankin, I. Zh. Bunin, and A. A. Oksogoev, *Synergetrics and Fractals in Materials Science* [in Russian] Nauka, Moscow (1994).
4. B. K. Barakhtin, Yu. I. Meshcheryakov, and G. G. Savenkov, "Dynamic and fractal properties of SP-28 steel under high-velocity loading," *Zh. Tekh. Fiz.*, **68**, No. 10, 43–49 (1998).
5. G. G. Savenkov, "Fractal-cluster model of spall fracture," *Zh. Tekh. Fiz.*, **72**, No. 12, 44–48 (2002).
6. S. A. Atroshenko, S. A. Gladyshev, and Yu. I. Meshcheryakov, " Structural level scale mechanisms for fracture of dynamically loaded media," in: *Proc. 4-th All-Union Conf. on Detonation*, Vol. 1, Telavi (1988), pp. 286–292.
7. Q. Y. Long, Li Suqin, and C. W. Lung, "Studies on the fractal dimension of the fracture surface formed by slow stable crack propagation," *J. Appl. Phys.*, **24**, 602–607 (1991).
8. É. V. Kozlov, "Parameters of the mesosrtucture and mechanical properties of single-phase metallic materials," *Vopr. Materialoved.*, No. 1. 50–69 (2002).
9. V. N. Aptukov, "Two stages of spallation," *Combust., Expl., Shock Waves*, No. 5, 631–636 (1985).
10. Yu. I. Fadeenko, "Temporal fracture criteria in solid-state dynamics," in: *Dynamics of Continuous Media* (collected scientific papers) [in Russian], No. 32, Inst. of Hydrodynamics, Sib. Div., Russian Acad. of Sci., Novosibirsk (1977), pp. 95–122.
11. V. I. Betehtin, A. I. Petrov, and A. G. Kadomtsev, "Life, development, and curing of microcracks in metals," in: *Physics of Strength and Plasticity* [in Russian], Nauka, Leningrad, (1986), pp. 41–48.

12. V. V. Novozhilov, “Necessary and sufficient criterion of brittle strength,” *Prikl. Mat. Mekh.*, **33**, No. 2, 212–222 (1969).
13. N. F. Morozov and Yu. V. Petrov, “Conception of structural time in dynamic fracture theory,” *Dokl. Akad. Nauk SSSR*, **324**, No. 5, 964–967 (1992).
14. N. F. Morozov, Yu. V. Petrov, and A. V. Utkin, “On the calculation of the limiting intensity of pulsed loads,” *Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela*, No. 5, 181–182 (1988).
15. A. G. Ivanov, “On possible causes of brittle fracture,” *J. Appl. Mech. Tekh. Fiz.*, No. 3, 439–442 (1988).
16. H.-O. Peitgen and P. H. Richter, *Beauty of Fractals: Images of Complex Dynamical Systems*, Springer-Verlag (1945).
17. R. L. Salganik, “Mechanics of bodies with a large number of cracks,” *Izv. Akad. Nauk SSSR. Mekh. Tverd. Tela*, No. 4, 149–158 (1973).
18. A. G. Ivanov (ed.), *Fracture of Objects of Different Scales in Explosion* [in Russian], Inst. of Exp. Phys., Sarov (2001).
19. V. V. Zosimov and L. M. Lyamshev, “Fractals in wave processes,” *Usp. Fiz. Nauk*, **165**, No. 4, 361–401 (1995).
20. A. A. Éfros, *Physics and Geometry of Disorder* [in Russian] Nauka, Moscow (1982).
21. G. G. Savenkov,“Deformation and fracture mechanisms of plastic and rigid bodies under high-velocity interaction,” Doct. Dissertation in Tech. Sci., Saint Petersburg (2003).